Combinatorics Examples

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May 2, 2026

Combinatorics

Binomial Theorem

Example 1. factorial function
Observe that $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$.

Example 2. binomial coefficient
Observe that $\binom{5}{3} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$.
Observe that $\binom{5}{3} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = \frac{5 \cdot 4}{2 \cdot 1} = 10$.

Example 3. binomial coefficient symmetry property
Since $\binom{7}{3} = \frac{7!}{4!3!} = \frac{7!}{3!4!} = \binom{7}{4}$, then $\binom{7}{3} = \binom{7}{4}$.

Example 4. binomial coefficient Pascal's Rule
Since $\binom{7}{4} = 35 = 20 + 15 = \binom{6}{3} + \binom{6}{4}$, then $\binom{7}{4} = \binom{6}{3} + \binom{6}{4}$.
This is easily visualized using Pascal's triangle.

Proposition 5. ***The number of subsets of a set of size*** n ***is*** 2^n***.***
For any positive integer n, $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Proof. Let n be any positive integer.

Then

$$
\begin{aligned}
2^n &= (1+1)^n \\
&= \sum_{k=0}^{n} \binom{n}{k} \cdot 1^{n-k} \cdot 1^k \\
&= \sum_{k=0}^{n} \binom{n}{k} \cdot 1 \cdot 1 \\
&= \sum_{k=0}^{n} \binom{n}{k} \cdot 1 \\
&= \sum_{k=0}^{n} \binom{n}{k}.
\end{aligned}
$$

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