General Math Notes

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Trig Facts and Identities

θ	$\sin(\theta)$	$\cos(\theta)$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0

Let $\theta \in \mathbb{R}$.
Then $\cos(\frac{\pi}{2} - \theta) = \sin(\theta)$.

Let $a, b \in \mathbb{R}$.
$\sin(a + b) = \sin a \cos b + \cos a \sin b$.
$\cos(a + b) = \cos a \cos b - \sin a \sin b$

Definition 1. Max/Min binary operations
Let $x, y \in \mathbb{R}$.
Define binary operations $\max : \mathbb{R}^2 \to \mathbb{R}$ and $\min : \mathbb{R}^2 \to \mathbb{R}$ by

$$\max(x, y) = x \vee y = \begin{cases} x, & x \geq y \\ y, & y \geq x \end{cases}$$

$$\min(x, y) = x \wedge y = \begin{cases} x, & x \leq y \\ y, & y \leq x \end{cases}$$

Let $x, y, z \in \mathbb{R}$.
Then $x \wedge (y \wedge z) = (x \wedge y) \wedge z$ (max is associative)

Definition 2. Convex Set
Let V be a vector space over field $\mathbb{R}$.
Let $S \subseteq V$.
Then S is **convex** iff $(\forall \vec{v}, \vec{w} \in S)(\forall t \in \mathbb{R}, 0 \leq t \leq 1)[t\vec{v} + (1 - t)\vec{w} \in S]$.

The closed interval $[0,1] \in \mathbb{R}^1$ is convex.

Let S_1 and S_2 be arbitrary convex sets in vector space V. Let $\vec{v}, \vec{w} \in S_1 \cap S_2$ be arbitrary. Let $t \in \mathbb{R}$ such that $t \in [0,1]$. Since $\vec{v} \in S_1 \cap S_2$, then $\vec{v} \in S_1$ and $\vec{v} \in S_2$. Since $\vec{w} \in S_1 \cap S_2$, then $\vec{w} \in S_1$ and $\vec{w} \in S_2$. Since S_1 is convex, then $t\vec{v} + (1-t)\vec{w} \in S_1$. Since S_2 is convex, then $t\vec{v} + (1-t)\vec{w} \in S_2$. Hence, $t\vec{v} + (1-t)\vec{w} \in S_1 \cap S_2$. Thus, the intersection of convex sets S_1 and S_2 is convex. Since S_1 and S_2 are arbitrary, then the intersection of convex sets S_1 and S_2 is convex for every S_1 and every S_2.

Therefore the intersection of any two convex sets in a vector space is convex.