# Multivariable Math Examples

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## Vectors

### Coordinate Vectors

**Example 1. vector addition**
Let $\vec{x} \in \mathbb{R}^4$ and $\vec{y} \in \mathbb{R}^4$ be vectors such that $\vec{x} = (2, -1, 0, 3)$ and $\vec{y} = (0, 7, -2, 3)$.
Compute $\vec{x} + \vec{y}$.

**Solution.** Observe that $\vec{x} + \vec{y} = (2, 6, -2, 6)$. □

**Example 2. scalar multiplication**
Let $\vec{x} \in \mathbb{R}^4$ be a vector such that $\vec{x} = (2, -1, 0, 3)$.
Compute $3\vec{x}$.

**Solution.** Observe that $3\vec{x} = 3(2, -1, 0, 3) = (6, -3, 0, 9)$. □

**Example 3. vector as a linear combination of basis vectors**
Let $\vec{v} \in \mathbb{R}^3$ be a vector such that $\vec{v} = (1, 2, -7)$.
Express $\vec{v}$ in terms of the basis vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

**Solution.** Observe that $\vec{v} = \vec{e}_1 + 2\vec{e}_2 - 7\vec{e}_3$. □

**Example 4. linear combination of vectors**
Let $\vec{v} \in \mathbb{R}^3$ be a vector such that $\vec{v} = (2, 3, 4)$.
Express $\vec{v}$ as a linear combination of the vectors $(1, 1, 1), (1, 1, 0), (1, 0, 0)$.

**Solution.** We must find scalars $a_1, a_2, a_3 \in \mathbb{R}$ such that $\vec{v} = a_1(1, 1, 1) + a_2(1, 1, 0) + a_3(1, 0, 0)$.

Suppose there exist scalars $a_1, a_2, a_3 \in \mathbb{R}$ such that $\vec{v} = a_1(1, 1, 1) + a_2(1, 1, 0) + a_3(1, 0, 0)$.

Then $(2, 3, 4) = a_1(1, 1, 1) + a_2(1, 1, 0) + a_3(1, 0, 0)$.

Observe that

$$\begin{aligned}
(2, 3, 4) &= a_1(1, 1, 1) + a_2(1, 1, 0) + a_3(1, 0, 0) \\
&= (a_1, a_1, a_1) + (a_2, a_2, 0) + (a_3, 0, 0) \\
&= (a_1 + a_2, a_1 + a_2, a_1) + (a_3, 0, 0) \\
&= (a_1 + a_2 + a_3, a_1 + a_2, a_1).
\end{aligned}$$

Hence, $(2,3,4) = (a_1+a_2+a_3, a_1+a_2, a_1)$, so $2 = a_1+a_2+a_3$ and $3 = a_1+a_2$ and $4 = a_1$.

Thus, $a_2 = 3 - a_1 = 3 - 4 = -1$, so $a_3 = 2 - a_1 - a_2 = 2 - 4 - (-1) = -1$.

Therefore, $a_1 = 4$ and $a_2 = -1$ and $a_3 = -1$, so $\vec{v} = (2,3,4) = 4(1,1,1) - 1(1,1,0) - 1(1,0,0)$. $\square$

## Geometry of Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$