Multivariable Math Notes

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Vectors

Coordinate Vectors

Definition 1. unit vector $\vec{e}_k$ in $\mathbb{R}^n$ vector space

Let $n \in \mathbb{Z}^+$.

Let $(\mathbb{R}^n, +, \cdot)$ be the n dimensional vector space over $\mathbb{R}$.

Define the vector $\vec{e}_k \in \mathbb{R}^n$ to be the n tuple in which the k^{th} coordinate is one and all other coordinates are zero.

Observe that $\vec{e}_1 = (1, 0, ..., 0)$ in $\mathbb{R}^n$.

Observe that $\vec{e}_2 = (0, 1, 0, ..., 0)$ in $\mathbb{R}^n$.

Observe that $\vec{e}_n = (0, ..., 0, 1)$ in $\mathbb{R}^n$.

Proposition 2. ***The set of vectors $\{\vec{e}_1, ..., \vec{e}_n\}$ is a natural basis for $\mathbb{R}^n$.***

Let $n \in \mathbb{Z}^+$.

Let $\vec{v} = (v_1, v_2, ..., v_n)$ be a vector in $\mathbb{R}^n$.

Then $\vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2 + ... + v_n\vec{e}_n$.

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Each scalar v_k is a **coordinate of the vector** $\vec{v}$ relative to the natural basis $\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\}$.

Since $\vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2 + ... + v_n\vec{e}_n$, then $\vec{v}$ **is a linear combination** of the basis vectors $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$.

Geometry of Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$