Multivariable Math Theory

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Vectors

Coordinate Vectors

Proposition 1. ***The set of vectors*** $\{\vec{e}_1, ..., \vec{e}_n\}$ ***is a natural basis for*** $\mathbb{R}^n$***.***

Let $n \in \mathbb{Z}^+$*.*

Let $\vec{v} = (v_1, v_2, ..., v_n)$ *be a vector in* $\mathbb{R}^n$*.*

Then $\vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2 + ... + v_n\vec{e}_n$*.*

Proof. Since $\vec{v} = (v_1, v_2, ..., v_n) \in \mathbb{R}^n$, then $v_1, v_2, ..., v_n \in \mathbb{R}$.

Observe that

$$\begin{aligned} v_1\vec{e}_1 + v_2\vec{e}_2 + ... + v_n\vec{e}_n &= v_1(1, 0, ..., 0) + v_2(0, 1, ..., 0) + ... + v_n(0, 0, ..., 1) \\ &= (v_1, 0, ..., 0) + (0, v_2, ..., 0) + ... + (0, 0, ..., v_n) \\ &= (v_1, v_2, ..., v_n) \\ &= \vec{v}. \end{aligned}$$

Therefore, $\vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2 + ... + v_n\vec{e}_n$. □

Geometry of Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$