Logic Examples

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Propositional Calculus

Example 1. $P \vee \neg P$ **is a tautology.**
Let P be a statement form.
Then $P \vee \neg P$ is a statement form.
When P is true, $P \vee \neg P$ is true, so $(P \vee \neg P) = T$.
When P is false, $P \vee \neg P$ is true, so $(P \vee \neg P) = T$.
Therefore, $P \vee \neg P$ is a tautology.
Hence, $P \vee \neg P \Leftrightarrow T$.

Example 2. $P \wedge \neg P$ **is a contradiction.**
Let P be a statement form.
Then $P \wedge \neg P$ is a statement form.
When P is true, $P \wedge \neg P$ is false, so $P \wedge \neg P = F$.
When P is false, $P \wedge \neg P$ is false, so $P \wedge \neg P = F$.
Therefore, $P \wedge \neg P$ is a contradiction.
Hence, $P \wedge \neg P \Leftrightarrow F$.

Example 3. The negation of a tautology is a contradiction.
Let P be a statement form.
Observe that

$$\begin{aligned} \neg(P \vee \neg P) &\Leftrightarrow \neg P \wedge \neg\neg P \\ &\Leftrightarrow \neg P \wedge P \\ &\Leftrightarrow P \wedge \neg P \end{aligned}$$

Therefore $P \wedge \neg P$ is a contradiction.
Hence, $P \wedge \neg P \Leftrightarrow F$.

Example 4. The negation of a contradiction is a tautology.
Let P be a statement form.
Observe that

$$\begin{aligned} \neg(P \wedge \neg P) &\Leftrightarrow \neg P \vee (\neg\neg P) \\ &\Leftrightarrow \neg P \vee P \\ &\Leftrightarrow P \vee \neg P \end{aligned}$$

Therefore $P \vee \neg P$ is a tautology.
Hence, $P \vee \neg P \Leftrightarrow T$.

Proofs

Example 5. fallacy of affirming the conclusion
$(P \Rightarrow Q) \wedge Q \vdash P$

Example 6. fallacy of denying the hypothesis
$(P \Rightarrow Q) \wedge \neg P \vdash \neg Q$