Number Theory Examples

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Divisibility and greatest common divisor

Miscellaneous Stuff

Example 1. The set $\{-12, -4, 11, 13, 22, 82, 91\}$ is a complete set of residues modulo 7.

Proof. Let $S = \{-12, -4, 11, 13, 22, 82, 91\}$.

To prove S is a complete set of residues modulo 7, we must prove each element of S is congruent modulo 7 to exactly one of the integers in $\{0, 1, 2, ..., 6\}$.

Observe that

$$\begin{aligned} 91 &\equiv 0 \pmod 7 \\ 22 &\equiv 1 \pmod 7 \\ -12 &\equiv 2 \pmod 7 \\ -4 &\equiv 3 \pmod 7 \\ 11 &\equiv 4 \pmod 7 \\ 82 &\equiv 5 \pmod 7 \\ 13 &\equiv 6 \pmod 7 \end{aligned}$$

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