Analysis Examples

Jason Sass

May 19, 2023

Metric spaces

Example 1. $(\mathbb{R}, d)$ **with Euclidean metric** d

The set of real numbers $\mathbb{R}$ with the distance function $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $d(x, y) = |x - y|$ for all $(x, y) \in \mathbb{R} \times \mathbb{R}$ is a metric space.

Proof. Let $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a function defined by $d(x, y) = |x - y|$ for all $(x, y) \in \mathbb{R} \times \mathbb{R}$.

Since $0 \in \mathbb{R}$, then $\mathbb{R} \neq \emptyset$, so $\mathbb{R}$ is not empty.

Since $\mathbb{R}$ is an ordered field, then $|x - y| = 0$ iff $x = y$ for all $x, y \in \mathbb{R}$ and $|x - y| = |y - x|$ for all $x, y \in \mathbb{R}$ and $|x - y| \leq |x - z| + |z - y|$ for all $x, y, z \in \mathbb{R}$.

Therefore, d is a metric on $\mathbb{R}$, so $(\mathbb{R}, d)$ is a metric space. □