Differentiation of real valued functions Calculus Exercises

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May 19, 2023

Derivative of a real valued function

Exercise 1. A manufacturer of lighting fixtures has daily production costs of $C = 800 - 10x + \frac{x^2}{4}$.

How many fixtures x should be produced each day to minimize costs?

Solution. The daily production cost C is a function of fixtures x produced each day.

Let $C : [0, \infty) \to \mathbb{R}$ be the function defined by $C(x) = 800 - 10x + \frac{x^2}{4}$.

Then $\frac{dC}{dx} = -10 + \frac{x}{2} = \frac{x}{2} - 10 = \frac{x-20}{2}$ for all $x \geq 0$.

We see that $\frac{dC}{dx} = 0$ iff $x = 20$.

Consider a δ neighborhood of 20.

Since C is a polynomial function, then C is continuous, so C is continuous on $(20 - \delta, 20 + \delta)$.

If $x < 20$, then $x - 20 < 0$, so $\frac{dC}{dx} < 0$.

Hence, $\frac{dC}{dx} < 0$ for all $x \in [0, 20)$.

Since $(20 - \delta, 20) \subset [0, 20)$, then $\frac{dC}{dx} < 0$ for all $x \in (20 - \delta, 20)$.

Thus, C is decreasing on the interval $(20 - \delta, 20)$.

If $x > 20$, then $x - 20 > 0$, so $\frac{dC}{dx} > 0$.

Hence, $\frac{dC}{dx} > 0$ for all $x \in (20, \infty)$.

Since $(20, 20 + \delta) \subset (20, \infty)$, then $\frac{dC}{dx} > 0$ for all $x \in (20, 20 + \delta)$.

Thus, C is increasing on the interval $(20, 20 + \delta)$.

Therefore, by the first derivative test, $C(20)$ is a relative minimum, so the cost C is minimized when $x = 20$.

We conclude that 20 fixtures should be produced each day to minimize the production costs. $\square$