Functions Examples

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Functions

Example 1. A function specified by listing its ordered pairs.
Let $f_1 = \{(2,3),(3,5),(4,7),(5,9)\}$.
Let $f_2 = \{(1,5),(1,-1),(4,7),(5,9)\}$.

Since f_1 is a set of ordered pairs, then f_1 is a relation.
Since no two distinct ordered pairs of f_1 have the same first element, then f_1 is a function.
The domain of f_1 is the set $\{2,3,4,5\}$.
The range of f_1 is the set $\{3,5,7,9\}$.

Since f_2 is a set of ordered pairs, then f_2 is a relation.
The domain of f_2 is the set $\{1,4,5\}$.
The range of f_2 is the set $\{-1,5,7,9\}$.
Since $(1,5) \in f_2$ and $(1,-1) \in f_2$, but $5 \neq -1$, then f_2 is not a function.

Example 2. A function specified by its domain and a rule of correspondence
Let f be defined by $dom f = \{x \in \mathbb{R} : x \geq 0\}$ and $f(x) = x^2$.
Then f is a function.

Example 3. equal functions
Let $f = \{(1,2),(2,4),(3,6)\}$.
Let g be a function such that $g = f$.
Then the domain of g is $dom g = dom f = \{1,2,3\}$.
Observe that $g(1) = f(1) = 2$, and $g(2) = f(2) = 4$, and $g(3) = f(3) = 6$.

Example 4. identity function
Let S be a set.
The **identity function on** S, denoted I_S, is defined by the rule $I_S(x) = x$ for all $x \in S$.

Example 5. constant function
Let S be a set.
Let $k \in S$.

The **constant function defined on** S, denoted C_S, is defined by the rule $C_S(x) = k$ for all $x \in S$.

Example 6. map

Let $f : [-\frac{\pi}{2}, \frac{\pi}{2}] \to \mathbb{R}$ be defined by $f(x) = \sin x$ for all $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

Then $f : [-\frac{\pi}{2}, \frac{\pi}{2}] \to \mathbb{R}$ is a map.

The function f is the sin function.

The domain of f is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

The codomain of f is $\mathbb{R}$.

The range of f is the closed interval $[-1, 1]$, a subset of the codomain.

Example 7. distinct maps can specify the same function

Let $f_1 : \mathbb{R} \to \mathbb{R}$ be defined by $f_1(x) = x^2$.

Let $f_2 : \mathbb{R} \to [0, \infty)$ be defined by $f_2(x) = x^2$.

Observe that f_1 and f_2 specify the same function, the square function.

Since $\mathbb{R} \neq [0, \infty)$, then the co-domain of f_1 and f_2 are not equal, so f_1 and f_2 are distinct maps.

Example 8. equal maps

Let $f : \{1, 2, 3, \} \to \mathbb{Z}^+$ be defined by $f(n) = 2n$ for all $n \in \{1, 2, 3\}$.

Let $g : \{1, 2, 3\} \to \mathbb{Z}^+$ be defined by $g(1) = 2$ and $g(2) = 4$ and $g(3) = 6$.

Then $f : \{1, 2, 3, \} \to \mathbb{Z}^+$ and $g : \{1, 2, 3, \} \to \mathbb{Z}^+$ are equal.

Proof. Since $f = \{(1, 2), (2, 4), (3, 6)\}$, then f is a relation.

The domain of f is the set $\{1, 2, 3\}$, so $domf = \{1, 2, 3\}$.

The range of f is the set $\{2, 4, 6\}$, so $rngf = \{2, 4, 6\}$.

Since $rngf = \{2, 4, 6\}$ and $\{2, 4, 6\} \subset \mathbb{Z}^+$, then $rngf \subset \mathbb{Z}^+$.

Since no two distinct ordered pairs of f have the same first element, then f is a function.

Since f is a function and $domf = \{1, 2, 3\}$ and $rngf \subset \mathbb{Z}^+$, then $f : \{1, 2, 3, \} \to \mathbb{Z}^+$ is a map.

Since $g = \{(1, 2), (2, 4), (3, 6)\}$, then g is a relation.

The domain of g is the set $\{1, 2, 3\}$, so $domg = \{1, 2, 3\}$.

The range of g is the set $\{2, 4, 6\}$, so $rngg = \{2, 4, 6\}$.

Since $rngg = \{2, 4, 6\}$ and $\{2, 4, 6\} \subset \mathbb{Z}^+$, then $rngg \subset \mathbb{Z}^+$.

Since no two distinct ordered pairs of g have the same first element, then g is a function.

Since g is a function and $domg = \{1, 2, 3\}$ and $rngg \subset \mathbb{Z}^+$, then $g : \{1, 2, 3, \} \to \mathbb{Z}^+$ is a map.

Since $f = \{(1, 2), (2, 4), (3, 6)\}$ and $g = \{(1, 2), (2, 4), (3, 6)\}$, then $f = g$.

The codomain of f and g is $\mathbb{Z}^+$, so f and g have the same codomain.

Since $f = g$, and f and g have the same codomain, then the maps $f : \{1, 2, 3, \} \to \mathbb{Z}^+$ and $g : \{1, 2, 3, \} \to \mathbb{Z}^+$ are equal. $\square$

Example 9. identity map on a set

Let S be a set.
Let $I_S : S \to S$ be a map defined by $I_S(x) = x$ for all $x \in S$.
We call I_S the **identity map on** S.

Let S be a set.
Let $I_S : S \to S$ be the identity map on S.
Then $I_S(x) = x$ for all $x \in S$.
The domain of I_S is S.
The codomain of I_S is S.
The range of I_S is S.

Therefore, I_S maps each element of S onto itself.
Since I_S is a bijective function on S, then I_S is a permutation map on S.

Example 10. constant map

Let S be a set.
Let $k \in S$.
Let $C_S : S \to S$ be a map defined by $C_S(x) = k$ for all $x \in S$.
We call C_S the **constant map on** S.

Let S be a set.
Let $k \in S$.
Let $C_S : S \to S$ be the constant map.
The domain of C_S is S.
The codomain of C_S is S.
The range of C_S is the set $\{k\}$, a subset of the codomain S.

Example 11. constant map on $\mathbb{R}$

Let $k \in \mathbb{R}$.
Let $C : \mathbb{R} \to \mathbb{R}$ be defined by $C(x) = k$ for all $x \in \mathbb{R}$.
Then $C : \mathbb{R} \to \mathbb{R}$ is the constant map defined on $\mathbb{R}$.
The domain of C is $\mathbb{R}$.
The codomain of C is $\mathbb{R}$.
The range of C is the set $\{k\}$, a subset of the codomain $\mathbb{R}$.

Example 12. restriction of a map

Let $f : \mathbb{R} \to \mathbb{R}$ be the map defined by $f(x) = x^2$.
Let $f|_S$ be the restriction of f to the interval $[0, \infty)$.
Then $f|_{[0,\infty)} : [0, \infty) \to \mathbb{R}$ is a map defined by $f|_{[0,\infty)}(x) = x^2$ for all $x \in [0, \infty)$.
The interval $[0, \infty)$ is a subset of $\mathbb{R}$, the domain of f.

Definition 13. inclusion map

Let $I_A : A \to A$ be the identity map on set A.
Let $S \subset A$.
Let $i_S : S \to A$ be the map defined by $i_S = (I_A)|_S$.
We call i_S the **inclusion map of** S **into** A.

Let $i_S : S \to A$ be the inclusion map of S into A.
Then $i_S = (I_A)|_S$, so the inclusion map is a restriction of the identity map.
Hence, $i_S(x) = x$ for all $x \in S$.
Therefore, i_S assigns to each element of S the same element, now in A.

Composition of functions

Example 14. Function composition is not commutative.
Let $f = \{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)\}$.
Let $g = \{(2,4),(3,5),(4,6),(5,7),(7,2)\}$.
Then f and g are functions, so $f \circ g$ and $g \circ f$ are functions.
Since $f \circ g = \{(2,5),(3,6),(4,7),(7,3)\}$ and $g \circ f = \{(1,4),(2,5),(3,6),(4,7),(6,2)\}$, then $f \circ g \neq g \circ f$.

Inverse functions

Example 15. inverse function as a set of ordered pairs
Let $f = \{(3,5),(5,8),(7,11),(9,14),(11,17)\}$.
Then f is a function and the inverse of f is $f^{-1} = \{(5,3),(8,5),(11,7),(14,9),(17,11)\}$.

Example 16. inverse of a map
Let $f : [0,\infty) \to \mathbb{R}$ be defined by $f(x) = x^2$.
Let $g : [0,\infty) \to \mathbb{R}$ by defined by $g(x) = \sqrt{x}$.
The map $g : [0,\infty) \to \mathbb{R}$ is the inverse of the map $f : [0,\infty) \to \mathbb{R}$.

Example 17. The inverse of the identity map on a set is the identity map on the set.
Let I_S be the identity map on a set S.
Then $I_S^{-1} = I_S$.

Proof. Let $g : S \to S$ be the map defined by $g(x) = x$ for all $x \in S$.
Then $g = I_S$.
Since $g \circ I_S = g = I_S$ and $I_S \circ g = g = I_S$, then g is the inverse of I_S.
Since $g = I_S$, then I_S is the inverse of I_S.
Therefore, $I_S^{-1} = I_S$. $\square$

Image and inverse image of functions